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ANALYSIS OF INDIVIDUAL DIFFERENCES IN MULTIDIMENSIONAL SCALING VIA AN N-WAY GENERALIZATION OF "ECKART-YOUNG" DECOMPOSITION

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3320 citations as of 4/1/2016

- **Individual Differences in Three-Way Data**
- **Weighted Euclidean Model & NILES Estimation**
- **Something Old, Something New, Something Borrowed, Something Blue**

Background: Three-Way Data in Psychology

Cattell was the first to consider the concept of three-way data.

See his **covariation chart (1952)**:

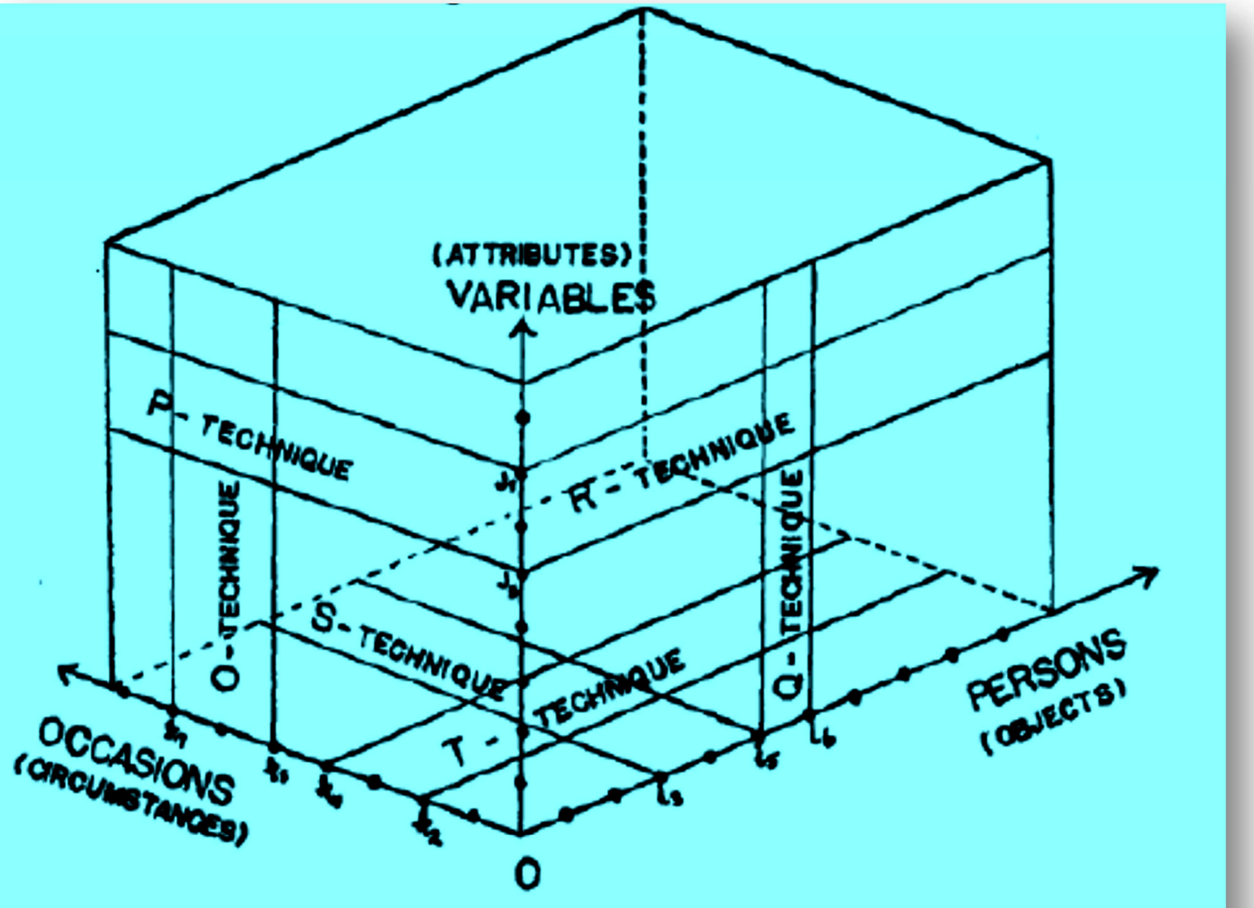
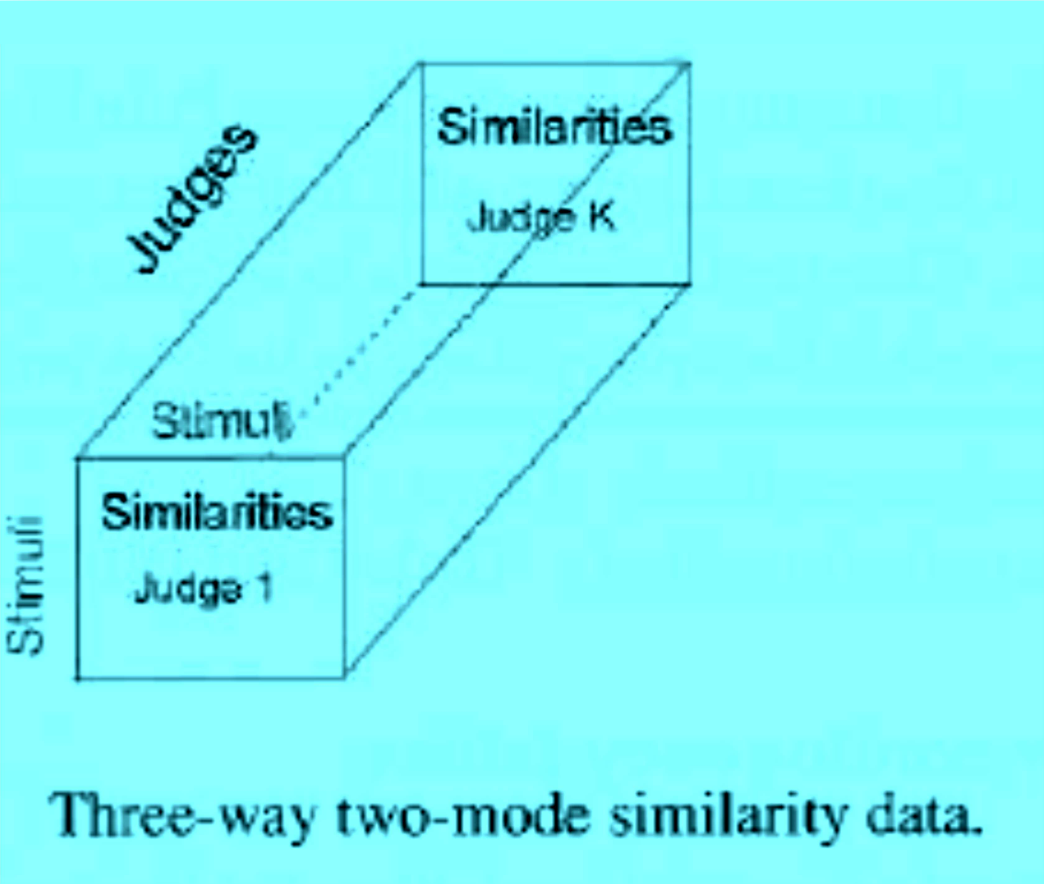


Fig. 1. The Covariation Chart

Factor analysis usually meant collapsing the data across one of the modes.

Carroll & Chang Considered Three-Way (Dis)Similarities

Three-way data in Multidimensional Scaling (MDS) usually consist of several **similarity** or **dissimilarity** matrices:



dissimilarities δ_{ijk} were usually **collapsed across individuals**:

$$\delta_{jk} = \frac{1}{m}\sum_{i=1}^{m}\delta_{ijk},$$

followed by MDS.

Points of View Analysis (Tucker & Messick, 1963)

Tucker (1960) had already developed a "**vector model**" for **two-way preference data** (= *Q*-factor analysis).

> "In the vector model, the multidimensional space represents the different viewpoints of the judges, each viewpoint being a one-dimensional scale [...]. In the distance model, the multidimensional space represents the different ways in which the stimuli are perceived to vary [...]. The present paper attempts to combine these two approaches by applying the vector model of stimulus scaling to measures of similarity between pairs of stimuli." (T&M, p. 336)

Results in **one separate MDS for each point of view** (obtained with FA + simple structure rotation).

Early application was **Helm & Tucker (1962) on color data** (identified color-blindness).

Weighted Euclidean Model (Horan, 1969)

C&C start their paper with some **criticisms** on PoV analysis:

> "Perhaps the most cogent criticism is that the method is little more powerful than doing separate scalings on the individual subjects."

Then they assume **one common space** X, but with different **saliences**, or importances:

$$d_{ijk} = \sqrt{\sum_{t=1}^{r} w_{it} \left(x_{jt} - x_{kt}\right)^2},$$

The individual spaces are:

$$y_{ijt} = w_{it}^{1/2} x_{jt}$$

➔Horan (1969) had proposed this model, where we note that it was submitted to *Psychometrika* in September **1964**, but published much later because the author was killed in a road accident. Bloxom (1968) also worked on it.

Classical MDS:
Young-Householder-Torgerson Transformation

Set up a matrix with elements $-\frac{1}{2}d_{ijk}^2$ and apply **double centering**. This transformation gives **scalar products**:

$$b_{ijk} = \sum_{t=1}^{r} y_{ijt} y_{ikt} = \sum_{t=1}^{r} w_{it} x_{jt} x_{kt}.$$

Arrived at this point, Horan had argued that we can estimate X by **averaging the scalar products** across individuals, and perform an **eigenvector/eigenvalue decomposition**. Horan did not estimate the weights.

Carroll & Chang Invoked a More General Model: CANonical DECOMPosition of 3-Way Tables

They switched from the model

$$b_{ijk} = \sum_{t=1}^{r} w_{it} x_{jt} x_{kt}$$ (INDSCAL model)

to a more general model:

$$z_{ijk} = \sum_{t=1}^{r} a_{it} b_{jt} c_{kt} \, .$$ (CANDECOMP model)

➔**Stroke of genius**: Solve a problem with **quadratic** terms by solving another model with **trilinear** terms. In addition, the Candecomp algorithm uses **Nonlinear Iterative Least Squares** (NILES, Wold, 1966).

NILES estimation (NIPALS, PLS, ALS)

Carroll & Chang credited Wold (1960):

> "This method of 'canonical decomposition' was suggested to us by a paper by Herman Wold (1966) […], in which a related method of decomposition of two-way tables was discussed […], involving what Wold calls a NILES (for "Nonlinear Iterative Least Squares") procedure. In the same paper […] Wold suggested the more general three-way model discussed here, but did not describe a computational scheme for this model (except for the special case of one dimension)."

However,

- From one- to more-dimensional merely requires **multiple** instead of **simple** regressions;
- Wold & Lyttkens (1969) gave **full-blown CANDECOMP**;
- Harshman (1970) had developed **PARAFAC (Parallel factors)**, with an identical algorithm by Bob Jennrich.

Do We Have to Give More Credit to Wold et al.?

Larry Hubert told me the following story about Herman Wold he heard from a well-known Stanford professor:

> "He said that Wold was not better cited because nobody wanted to give him any more encouragement than necessary. Apparently, (multivariate) statisticians who saw him walking down the street, would cross to the other side to avoid having to talk to him -- Wold used to go on and on about his work, whether the listener was interested or not."

Note that Wold (1966) was published in the first Krishnaiah volume, as was the Shepard & Carroll (1966) paper about **parametric mapping** (**PARAMAP**). Maybe either Shepard and/or Carroll met Wold at the meeting in Ohio in 1965?

Do We Have to Give More Credit to Horan?

When asked about this, Doug Carroll used to say

- that Horan **did not solve for the individual differences** (the weights);
- that his solution for the common space **did not solve the rotation problem**, while the INDSCAL algorithm does.

True, but the Carroll & Chang paper was worded in such a way the many readers got the impression that it proposed a **new model**, which was **not the case**.

Clearly, Horan was not able to emphasize the strong points of the weighted Euclidean model as much as Doug did.

Do We Have to Give More Credit to Richard Harshman?

Well, the **three-way community** already does so. They see Harshman as someone with more contributions to **three-way three-mode** models & algorithms, while Carroll had more contributions to **three-way two-mode** models & algorithms.

- Instead of Candecomp they talk about the Parafac or CP: the **Canonical Polyadic** decomposition, and refer to Hitchcock (1927) as the originator;
- The strong point of INDSCAL (unique orientation of axes) is much better understood from the starting point of Cattell (1944) picked up by Harshman in PARAFAC: the **Principle of Parallel Proportional Profiles**.

Prototypical Programmatic Paper

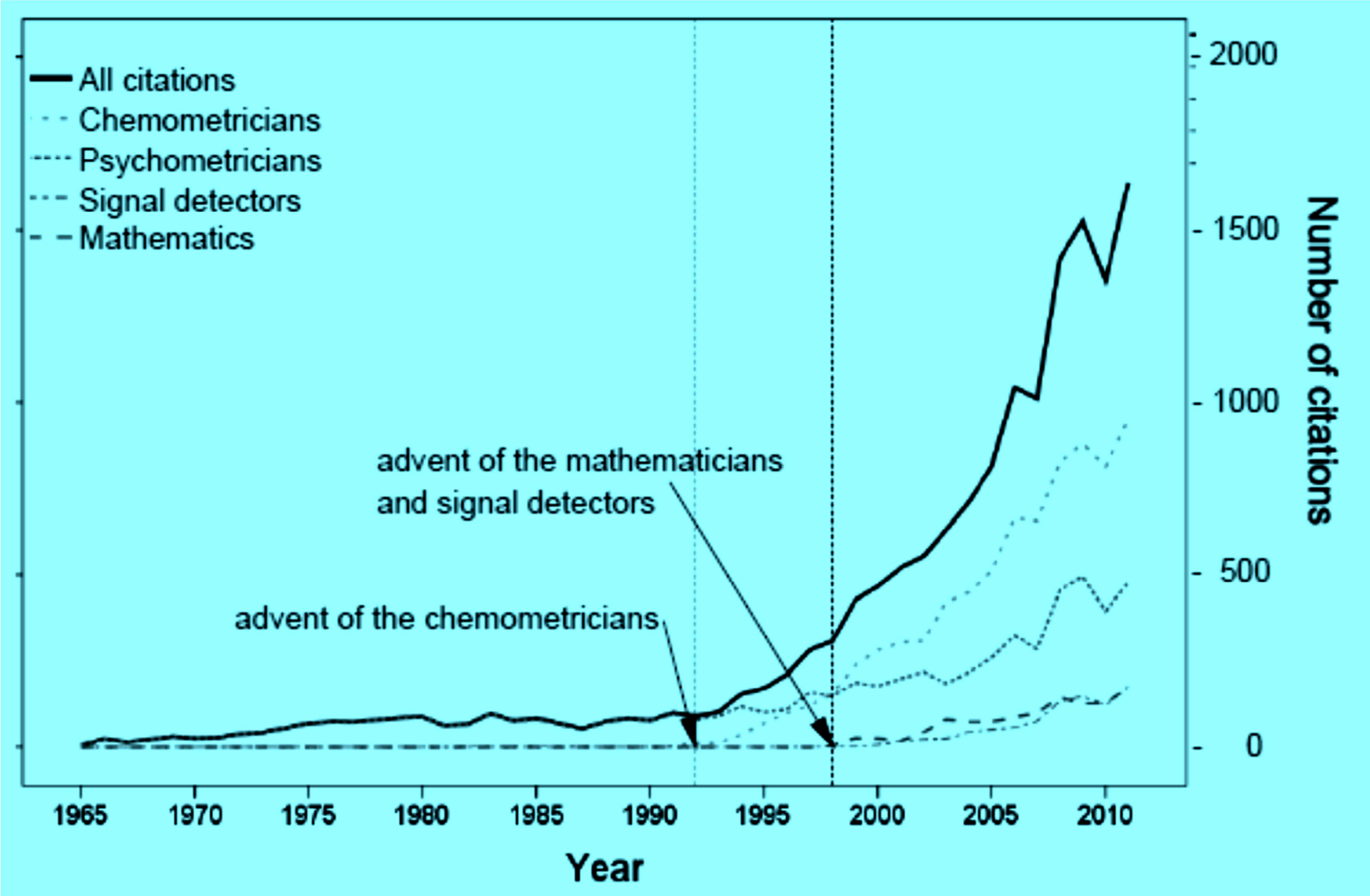
The larger part consisted of

- Extensive descriptions of two **illustrative datasets**;
- Persistent discussions of the strong points of the model (**unique orientation** of axes, interpretation of **weights**);
- Detailed comparisons, possible **modifications/extensions**.

It generated a lot of follow-up work,

- Technical, for example **CANDELINC** (Carroll, Pruzansky & Kruskal, 1980) and theoretical;
- Software (INDSCAL program was widely distributed) and many **applications** in variety of areas (marketing, sensometrics, chemometrics, signal detection,…).

The rise of Three-Way Citations (Kroonenberg, 2014)



<u>Psychometricians counted</u>: Carroll, Harshman, Kiers, Kroonenberg, Kruskal, Ten Berge, Tucker.

<u>Chemometricians counted</u>: Appellof, Bro, Smilde, …

Doug Carroll: the Navigator